UCB Math 128A, Spring 2014: Programming Assignment 1

Due February 25

In this assignment, we will address two issues with the Bisection method and Newton's method:

- Finding an interval $[a, b]$ for the Bisection method, with $f(a)$ and $f(b)$ having different signs.
- Combining the excellent convergence properties of Newton's method with the guaranteed root-finding of the Bisection method.

1. Implement a MATLAB function `findbracket` of the form

```
function [a,b]=findbracket(f,x0)
```

which finds an interval $[a, b]$ around x_0 such that $f(a)$ and $f(b)$ have different signs. Use the following strategy:

1. Start with $a = b = x_0$, and $dx = 0.001$
2. Subtract dx from a, and terminate if $f(a)f(b) < 0$
3. Add dx to b, and terminate if $f(a)f(b) < 0$
4. Multiply dx by 2 and repeat from step 2.

2. Implement a MATLAB function `newtonbisection` of the form

```
function p=newtonbisection(f,df,a,b,tol)
```

combining Newton's method and the Bisection method according to the following strategy:

1. Start with $p = a$
2. Attempt a Newton step $p = p - f(p)/f'(p)$
3. If p is outside of $[a, b]$, set $p = (a + b)/2$
4. If $f(p)f(b) < 0$, set $a = p$, otherwise set $b = p$
5. Terminate if $|f(p)| < \text{tol}$
6. Repeat from step 2.

Use the functions `newton` and `bisection` on the course web page as a starting point, this function will be like a combination of the two.

3. Run your function `newtonbisection` using $f(x) = \sin x - e^{-x}$ on the interval $[1.9, 30]$:

```
f=@(x) sin(x)-exp(-x);
df=@(x) cos(x)+exp(-x);
x=newtonbisection(f,df,1.9,30,1e-8);
```

Present the result in a table showing for each iteration the method used (Newton or Bisect), a, b, p, and $f(p)$.

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4. Use your combined `findbracket` and `newtonbisection` to solve for the roots of $f(x) = \sin x - e^{-x}$ with $x_0 = -3, -2, \ldots 10$:

```
f=@(x) sin(x)-exp(-x);
df=@(x) cos(x)+exp(-x);
for x0=-3:10
  [a,b]=findbracket(f,x0);
  x=newtonbisection(f,df,a,b,1e-8);
  [x0,a,b,x]
end
```

Present your results in a table showing x_0, a, b, and x.

Reporting requirements:

The GSIs will *not* run any submitted MATLAB codes. Prepare a report showing the requested information, which is essentially just your MATLAB functions and the computed tables. Give brief comments if things do not work as expected.