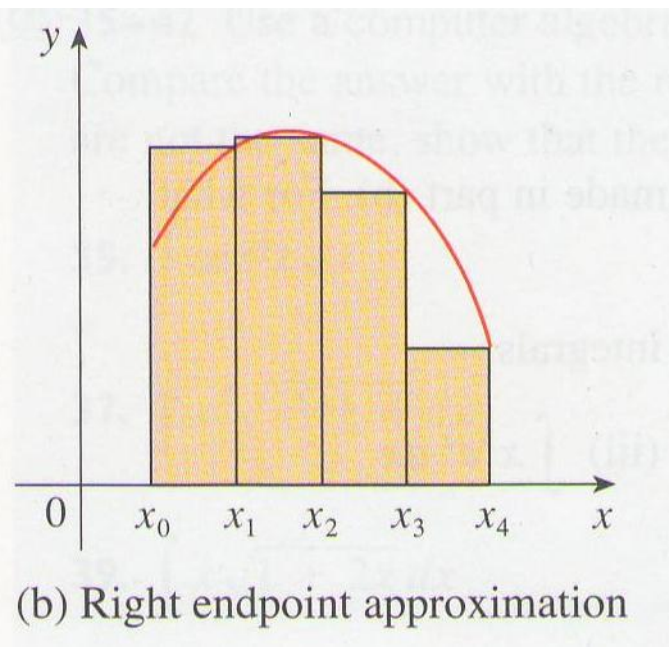


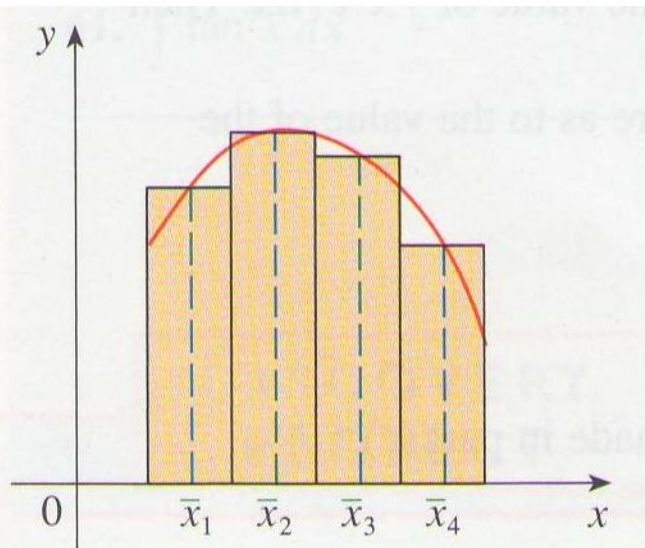
Left Endpoint Approximation

$$\int_a^b f(x) dx \approx L_n = \sum_{i=1}^n f(x_{i-1}) \Delta x$$



Right Endpoint Approximation

$$\int_a^b f(x) dx \approx R_n = \sum_{i=1}^n f(x_i) \Delta x$$



(c) Midpoint approximation

Midpoint Rule

$$\int_a^b f(x) dx \approx M_n = \Delta x [f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)]$$

$$\Delta x = \frac{b - a}{n}$$

$$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1}, x_i]$$

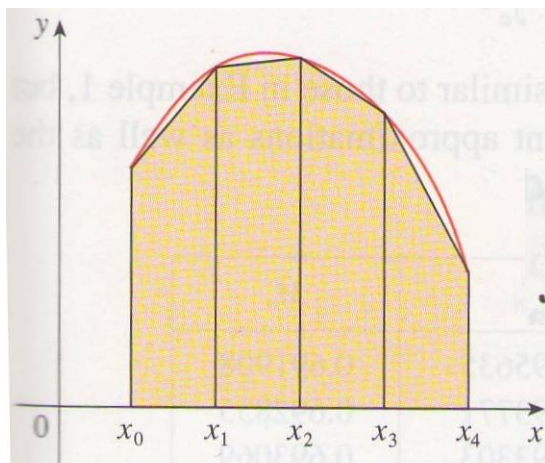


FIGURE 2

Trapezoidal approximation

Trapezoidal Rule

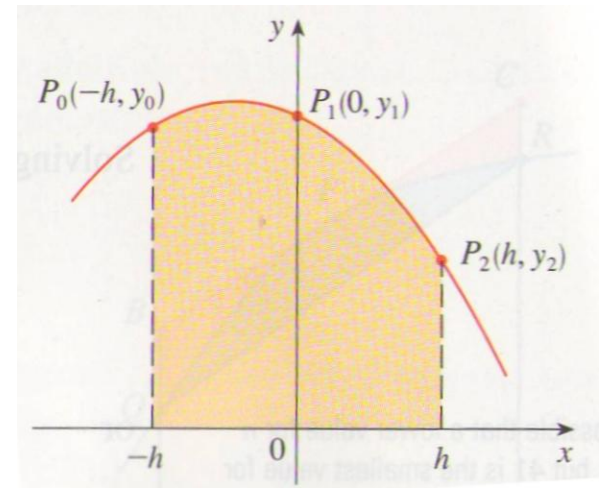
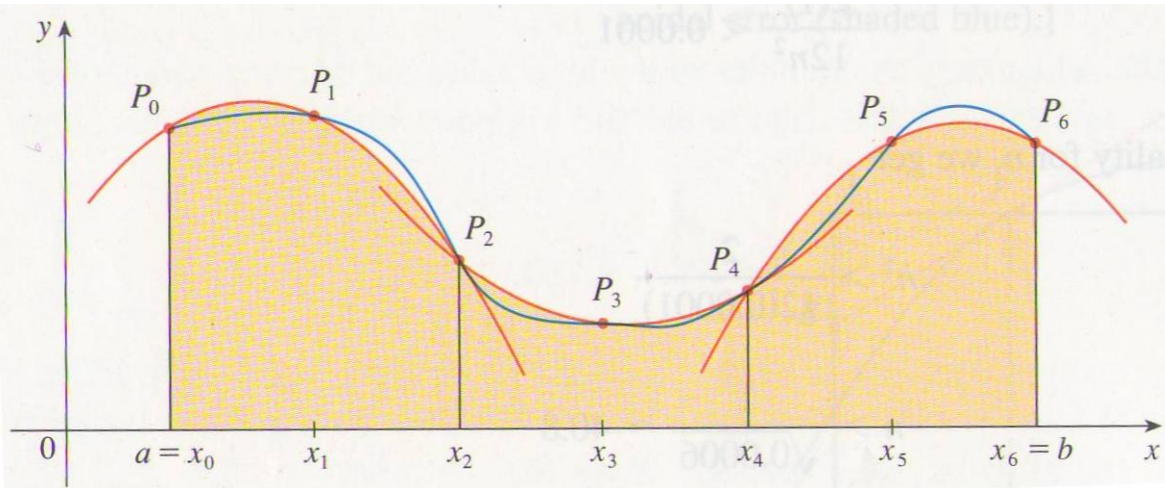
$$\int_a^b f(x) dx \approx T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

where $\Delta x = (b - a)/n$ and $x_i = a + i \Delta x$.

Simpson's Rule

$$\int_a^b f(x) dx \approx S_n = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

where n is even and $\Delta x = (b - a)/n$.



Error Bounds

3 ERROR BOUNDS Suppose $|f''(x)| \leq K$ for $a \leq x \leq b$. If E_T and E_M are the errors in the Trapezoidal and Midpoint Rules, then

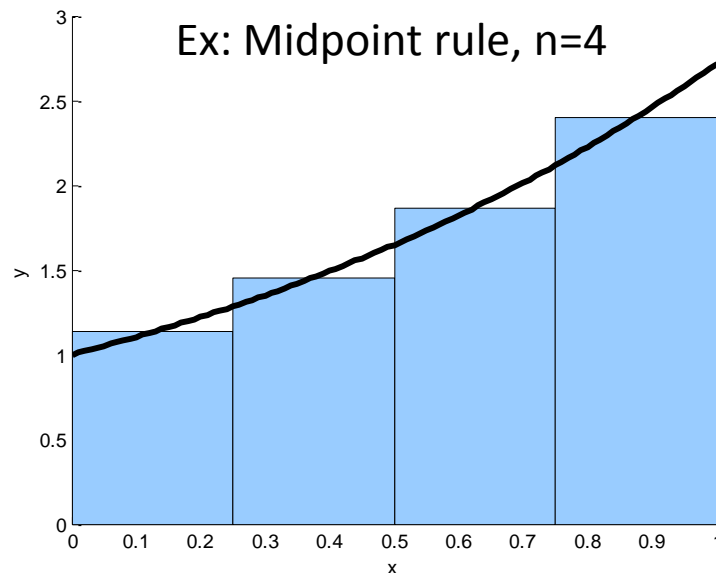
$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \quad \text{and} \quad |E_M| \leq \frac{K(b-a)^3}{24n^2}$$

4 ERROR BOUND FOR SIMPSON'S RULE Suppose that $|f^{(4)}(x)| \leq K$ for $a \leq x \leq b$. If E_S is the error involved in using Simpson's Rule, then

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$

$$\int_0^1 e^x dx = e - 1$$

$$\approx 1.718281828$$



	n	Left End	Right End	Midpoint	Trapezoidal	Simpson	Exact
<i>Integrals</i>	2	1.32436064	2.18350155	1.70051272	1.75393109	1.71886115	1.71828183
	4	1.51243668	1.94200713	1.71381528	1.72722190	1.71831884	1.71828183
	8	1.61312598	1.82791121	1.71716366	1.72051859	1.71828415	1.71828183
	16	1.66514482	1.77253744	1.71800219	1.71884113	1.71828197	1.71828183
	32	1.69157351	1.74526981	1.71821191	1.71842166	1.71828184	1.71828183

	n	Midpoint		Trapezoidal		Simpson	
	n	Bound	Actual	Bound	Actual	Bound	Actual
<i>Errors</i>	2	0.02831544	0.01776911	0.05663087	0.03564926	0.00094385	0.00057932
	4	0.00707886	0.00446655	0.01415772	0.00894008	0.00005899	0.00003701
	8	0.00176971	0.00111816	0.00353943	0.00223676	0.00000369	0.00000233
	16	0.00044243	0.00027964	0.00088486	0.00055930	0.00000023	0.00000015
	32	0.00011061	0.00006992	0.00022121	0.00013983	0.00000001	0.00000001