High-Order Discontinuous Galerkin Simulation of Flapping Wings

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1 Motivation

2 Problem Statement

3 High-Order Discontinuous Galerkin Solver

4 Results
Bio-Inspiration for Flapping Wing MAVs

- Develop high-order accurate simulation capabilities that capture the complex physics in flapping flight
- Use the computational tools for increased understanding and to design optimized flapping kinematics
High-Order Discontinuous Galerkin Simulations

- Complex flow features $\rightarrow$ High-order accuracy required
- Complex geometries, adaptation $\rightarrow$ Unstructured grids required
- Discontinuous Galerkin (DG) methods have these properties:

<table>
<thead>
<tr>
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<th>FVM</th>
<th>FDM</th>
<th>FEM</th>
<th>DG</th>
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</thead>
<tbody>
<tr>
<td>1) High-order/Low dispersion</td>
<td>$\times$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
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<td>2) Unstructured meshes</td>
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<td>$\times$</td>
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<td>3) Stability for conservation laws</td>
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- However, several problems to resolve:
  - High CPU/memory requirements (compared to FVM or H-O FDM)
  - Low tolerance to under-resolved features
  - High-order geometry representation and mesh generation

- The challenge is to make DG competitive for real-world problems
Outline

1. Motivation
2. Problem Statement
3. High-Order Discontinuous Galerkin Solver
4. Results
Flapping Motion

- Explicit analytical expression for the flapping motion
- Not an optimized flapping strategy, but representative and adequate for study of the computational models
- Prescribed symmetric wing motion with amplitude $A_\phi = 30^\circ$, angular frequency $\omega = 2\pi/20$, and flapping/twist angles:
  \[
  \phi(t) = A_\phi \cos \omega t \\
  \theta(t) = \varepsilon (a(X) \cos \omega t + b(X) \cos \omega t)
  \]
- Coefficients $a(X), b(X)$ chosen to align wing with flow
- Parameter $\varepsilon \in [0, 1]$ controls amount of feathering
Elliptical planform, chord at centerline $c = 1$, tip-to-tip span $b = 10$

HT13 airfoil for the entire wing span

Maximum wing thickness $t = 0.065$ at the wing centerline

Free-stream Mach number 0.1, Reynolds number 3,000

Various free-stream angles of attack: $0^\circ, 5^\circ, 10^\circ$
Flapping Motion, $\varepsilon = 0.5$
1. Motivation

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The 3DG Discontinuous Galerkin Solver

- High-order discretizations on unstructured meshes
- Capable of simulating challenging problems:
  - complex real-world geometries
  - transitional flows, multiple scales
  - moving and deforming domains
  - fluid-structure interactions
- General multiphysics framework applicable to a wide range of challenging problems

Unsteady Flows

Aeroacoustics

Thin Structures
The DistMesh Mesh Generator

  1. Start with any topologically correct initial mesh
  2. Move nodes to find force equilibrium in edges
     - Project boundary nodes using implicit geometry $\phi(x)$
     - Update element connectivities with Delaunay

Current impact of DistMesh:
- 99 journal citations to original publication
- 5,200 hits on Google
- Rewritten in C, C++, C#, Fortran 77/90, Python, Mathematica, Octave
- Used in books and in numerous courses
**Tetrahedral Mesh**

- Surfaces triangulated in parametric form using DistMesh, tetrahedra from Delaunay refinement with local improvements
- 231,000 tetrahedra, 4.6M high-order nodes in the half-domain
- Resolution focused around wing and in the wake
- Elements close to wing curved using nonlinear elasticity approach

[Persson/Peraire ’09]
Automatic generation of non-inverted curved elements largely an unresolved problem.

In general this is a global problem, affecting many elements except for simple isotropic 2-D meshes.

In [Persson/Peraire ’09], we proposed a non-linear solid mechanics approach, where the mesh is considered an elastic deformable solid.
Tetrahedral Mesh of Falcon Aircraft

- Real-world mesh with coarse but realistic elements
- Unstructured Delaunay refinement mesh, with highly curved boundary segments
- Many elements would invert with a local element-wise approach

Tetrahedral mesh

Elements with $I < 0.5$
Mesh Deformation for Moving Domains

- Non-linear solid mechanics approach also used to deform meshes
(Reed/Hill 1973, Lesaint/Raviart 1974, Cockburn/Shu 1989-, etc)

Consider non-linear hyperbolic system in conservative form:

\[ u_t + \nabla \cdot F_i(u) = 0 \]

Galerkin formulation:

\[
\int_{\kappa} [(u_h)_t] v_h \, dx - \int_{\kappa} F_i(u_h) \nabla v_h \, dx + \int_{\partial\kappa} \hat{F}_i(u_h^+, u_h^-, \hat{n}) v_h^+ \, ds = 0
\]

with \textit{numerical flux function} \( \hat{F}_i(u_L, u_R, \hat{n}) \)

Global problem: Find \( u_h \in V_h^p \) such that this weighted residual is zero for all \( v_h \in V_h^p \)

Error = \( O(h^{p+1}) \) for smooth solutions
The Compact Discontinuous Galerkin Method

- The CDG method: Efficient and practical scheme for viscous terms [Peraire/Persson ’08]
- Provably optimal accuracy $O(h^{p+1})$
- Higher stability/accuracy than LDG/BR2
- Sparsest known scheme (incl LDG/BR2/IP)
Parallel Newton-Krylov Solvers

- Implicit solvers required because of CFL restrictions from viscous effects, unstructured meshes, low Mach numbers, etc.
- Storage of Jacobian $J = \frac{\partial R}{\partial U}$ requires about 55GB of memory.
- Use block-ILU(0) preconditioner with MDF ordering [Persson ’08].
- Parallelize by domain decomposition and ILU preconditioner.
- Close to perfect speedup for time accurate simulations.
Example: ILES at $Re = 60,000$

- Implicit Large Eddy Simulations for flow past airfoil
- Separation and transition well captured
- Vortical structures: iso-surfaces of q-criterion ($\nabla^2 p/2\rho$)
Example: ILES at $Re = 60,000$

- Good agreement with XFOil and previously published ILES [Galbraith/Visbal ’08]

Average pressure and skin friction coefficients
ALE Formulation for Deforming Domains

- Use mapping-based ALE formulation for the moving domain [Visbal/Gaitonde ’02, Perssson/Bonet/Peraire ’08]
- Map from reference domain $V$ to physical deformable domain $v(t)$
- Introduce the *mapping deformation gradient* $G$ and the *mapping velocity* $v_X$ as

\[
G = \nabla_X G \quad \text{and} \quad v_X = \frac{\partial G}{\partial t} \bigg|_X
\]

and set $g = \det(G)$

- Transform equations to account for the motion
Mapping-based formulation gives arbitrarily high-order accuracy in space and time.
Example: ILES simulation of Heaving Airfoil

- SD7003 foil, pure heaving motion at 4 degrees AoA and Re=60k
Flapping Wing Mapping Function

- Need analytical expression that deforms the wing according to the angles $\phi(t), \theta(t)$, but extends smoothly to the entire domain.
- Our approach: Two shear motions, scaling to preserve wing area.

$$x(X, t) = \begin{bmatrix} X \cos \phi \\ Y \cos \theta \\ X \sin \phi + Y \sin \theta + Z \sec \phi \sec \theta, \end{bmatrix}$$

- Volume-preserving deformation gradient ($\det(G) = 1$):

$$G = \frac{\partial x}{\partial X} = \begin{bmatrix} \cos \phi & 0 & 0 \\ G_{21} & \cos \theta & 0 \\ G_{31} & G_{32} & \sec \phi \sec \theta \end{bmatrix}$$

- Grid velocity $\partial x/\partial t$ also found by symbolic differentiation.
Mapping Function
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High-Order Simulations

- Six test cases: AoA $\alpha = 0^\circ, 5^\circ, 10^\circ$, twist multiplier $\varepsilon = 1.0, 0.5$
- Polynomial order $p = 3$, 23 million DOFs
- 3-stage, 3rd order A-stable Diagonally Implicit Runge-Kutta in time
- 3 full flapping cycles, 600 implicit timesteps
- 32 computing nodes = 256 processes, run time about 3 days
- Visualization by Mach number (color) on isosurface of entropy

$\text{AoA } \alpha = 5^\circ$, twist multiplier $\varepsilon = 0.5$
Comparison with Potential Flow Solver

- Panel Method Potential Flow Solution
  - Efficient for large wing motions
  - Requires surface meshes only
- Inviscid, will not be able to model:
  - Flow separation
  - Leading edge vortices
  - Viscous drag
- Implementation Details
  - Linear basis unstructured panel method
  - Unsteady free vortex particle/sheet wakes
  - Accelerated iterative solution (pFFT, FMM)
Angle of Attack $\alpha = 0^\circ$, Twist Multiplier $\varepsilon = 1.0$

- Feathering – all forces small
- Mainly viscous forces $\rightarrow$ large relative errors with panel code

N-S lift (green), N-S drag (blue), Panel (dashed)
Angle of Attack $\alpha = 5^\circ$, Twist Multiplier $\varepsilon = 1.0$

- Lift production, some minor separation inboard
- Panel code over-predicts lift and thrust

N-S lift (green), N-S drag (blue), Panel (dashed)
Similar to $5^\circ$ case, but more flow separation

The unsteady flow gives significant noise in forces

N-S lift (green), N-S drag (blue), Panel (dashed)
Angle of Attack $\alpha = 0^\circ$, Twist Multiplier $\varepsilon = 0.5$

- Thrust production, symmetric w.r.t. up/down stroke
- The flow stays attached inboard, LEVs shed outboard into wake

N-S lift (green), N-S drag (blue), Panel (dashed)
Angle of Attack $\alpha = 5^\circ$, Twist Multiplier $\varepsilon = 0.5$

- Good force and lift production, but significant separation.
- Only slight over-prediction of lift/thrust by panel code.

N-S lift (green), N-S drag (blue), Panel (dashed)
Angle of Attack $\alpha = 10^\circ$, Twist Multiplier $\varepsilon = 0.5$

- Similar to $5^\circ$ case, but more flow separation
- Some unsteadiness in forces during downstroke
Grid Convergence ($\alpha = 5^\circ$, $\varepsilon = 0.5$)
Conclusions and Summary

Important steps toward practical high-order solver:

- Curved mesh generation using nonlinear elasticity
- Efficient viscous discretization (the CDG method)
- General purpose parallel multigrid/block ILU preconditioner
- Optimal accuracy for deformable domains by mapping approach

Higher fidelity methods critical for separated flows

Current work includes:

- New lower-cost discretizations and solvers
- More complex geometries/flight kinematics
- Optimal design of flapping wings
- Full fluid-structure coupling